History of mathematics in Norwegian textbooks

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In 1997, Norwegian authorities decided that history of mathematics should be an important part of mathematics education for Norwegian children (6-16 years old).[1] I have studied all Norwegian textbooks that have been approved by Norwegian authorities since the 1997 reform, and will summarise my main findings in this paper. The reader will perhaps find it interesting to contrast the situation in Norway with the situation in his/her own country.

Background[2]

Textbooks published before 1997 included very little history of mathematics. Norwegian teachers had (and have) little knowledge of history of mathematics. Therefore, textbook writers had fairly little to build on when they were facing the task of writing the new textbooks. The main interest in this article is to find out what choices were made.

The goals

The six main goals for mathematics education in Norway are:

- that pupils develop a positive attitude to mathematics, experience that it is meaningful and build confidence in their abilities in mathematics.
- that mathematics becomes a tool that pupils may use in school, leisure, work and society.
- that pupils are stimulated to use their fantasy, their resources and their knowledge to find solutions through investigative and problem solving activities and conscious choices of tools.
- that pupils develop skills in reading, formulating and communicating subjects and ideas where use of mathematical language and symbols is natural.
- that pupils develop insight in basic concepts and methods in mathematics, and develop their ability to see connections and structures and be able to understand and use logical reasoning and draw conclusions.
- that pupils develop insight in the history of mathematics and mathematics' role in culture and science.[3]

In addition to these "general" goals, there are specific goals for every grade.

Quantity

Of 15623 pages of mathematics in pupils' textbooks, there are about 193,6 pages of history of mathematics (these numbers are the total number of pages for all grades and all different textbooks). In average a pupil will meet a total of four pages of history of mathematics in his first four school years (ages 6-10), 19 pages in his next three years (ages 10-13), and at last about 13 pages in his last three years (ages 13-16). In all, the "average pupil" will meet about 36 pages of history of mathematics in his ten years in school.

Themes

There are five or six alternative sets of textbooks in each grade. The following table shows the subjects from the history of mathematics that are mentioned with more than one sentence in more than one textbook. The number in parenthesis shows the number of textbooks in that

[1] For my summary of the most important reasons for including history of mathematics in mathematics education, see http://www.hifm.no/~matematikk/ansatte/bjorns/mattehist/reasons.htm.
[2] See my article "The introduction of history of mathematics in Norwegian schools" for more information on the background. The article is included in the forthcoming Proceedings of the HPM 2000 in Taipei.
[3] This is my own translation from Norwegian.

grade that mentions the subject. The last column shows the number of themes that are mentioned in only one textbook in that grade.

Grade	Themes	Other themes
1 (age 6-7)	none	0
2 (age 7-8)	none	1
3 (age 8-9)	roman numerals (5), other numeral systems (3)	4
4 (age 9-10)	roman numerals (2)	2
5 (age 10-11)	old measuring systems (3), the calendar (3), roman numerals (2), pyramids (2)	4
6 (age 11-12)	roman numerals (5), hindu-arabic numerals (5), egyptian numerals (4), old measuring systems (4), mayan numerals (3), babylonian numerals (2)	22
7 (age 12-13)	numeral systems (5), Archimedes (2)	12
8 (age 13-14)	numeral systems (5), old measuring systems (3), calculation of π (2), the abacus (2)	7
9 (age 14-15)	Pythagoras (5), the golden ratio (3), pyramids (3), geometrical patterns (2)	12
10 (age 15-16)	numeral systems (4), the symbol "0" (2), architecture (2), number mysticism (2), indian mathematical problems (2), Archimedes (2), Pythagoras (2), Fibonacci (2) and Diophantes (2).	24

This table seems to suggest that there is little agreement among the writers as to which themes are the most important ones.

Errors

Being constructive is difficult; criticising is easy. Therefore I will go on criticising by giving examples of errors in Norwegian textbooks. I have divided the errors into five categories, as follows:

Errors that may strengthen misconceptions:

Luckily, there are not very many errors in this category. I will mention two:

- "Roman numerals are not really *tall*, they are letters symbolising *tall*". The Norwegian word "tall" can mean both number and numeral, but either way this is nonsensical. Roman numerals are no less *tall* than arabic numerals, even though we are less familiar with roman numerals.
- The calculation "V+V+V=III·V=XV=15" shows how difficult roman numerals can be if you want them to. This, in addition to claims that roman numerals are difficult to use in calculations make Romans seem stupid for using this system. What should be mentioned (and is mentioned in other books) is that Romans didn't need roman numerals in calculations, as they had the abacus.[4]

Myths:

One great danger in history of mathematics is that it will only be a retelling of myths. There are not very many examples of this in the textbooks, the most obvious example is this:

- "In 212 B. C. [Archimedes] was killed by a roman soldier while solving geometrical problems in the sand".

Anachronisms:

[4] In fact, using roman numerals in calculations would be more expensive, as it would take some material to write on, and something to write with.

Exercises that confuse modern and old notation are very common, for instance "X+X=". I'm not sure how serious this is, but it may support the misconception that Romans (for instance) did everything just like us, except that they used different symbols for their numbers.

Less important errors:

There are lots of errors when it comes to details – who created the square root symbol, who was the first to use parenthesis etc. These errors will probably go unnoticed by both pupils and teachers, and no one will be harmed. But it would be better if the information was correct.

Simplifications/inaccuracies:

Of course it is necessary to make some simplifications when trying to write about the history of mathematics to children. A little too often, these simplifications can also be labelled inaccuracies:

- "In mathematics, this number is called pi and written π. This was first described by W. Jones 1706 in an English textbook". The fact that the number pi has fascinated mathematicians for thousands of years (even if they didn't call it "pi") is obscured here.
- "This interesting 'triangle' of numbers was created by a French mathematician, Blaise Pascal, who lived in the 17th century. Therefore it is called 'Pascal's triangle'". Again, pupils might find it interesting to know that this triangle was studied long before Pascal.

Exercises

I will quote and comment on some of the exercises pupils are faced with when reading Norwegian textbooks:

- "Use reference books and the Internet to find names of mathematicians that worked with calculations of probabilities in the 17th and 18th centuries". I don't see what knowledge pupils will get out of this, and I don't see why the names of the mathematicians are particularly interesting.
- "The mathematician Blaise Pascal was one of the first to work with calculations of probabilities. He was born in 1623 and died in 1662. a) How old was Pascal when he died? b) How many years ago were Pascal born? c) How many years ago did Pascal die?"[5] These exercises are given to 16-year old pupils. I seriously doubt that they have a need to practice subtraction of four-digit numbers. I also doubt that they will feel that the exercises are meaningful. Moreover, reducing history of mathematics to calculations with numbers is a dehumanisation of history of mathematics – the opposite of what we wanted to do.
- "Now we will see how Egyptians calculated with their hieroglyphs. Egyptians added and subtracted in the same way that we do. But they multiplied in another way:

26·41	
13·82	82
12·82	
6·164	
3·328	328
2·328	
1·656	656
	1066

Try to explain what the old Egyptians did. Multiply other numbers in the same way. Can we use the same method when we divide?" At least, pupils will see that there are more than one way to do multiplication. Explaining how the methods work is an exercise in the

[5] This example is (sadly) not unique.

distributive law. Moreover, they will see that different cultures often do things in different ways. In my opinion, this is a meaningful exercise.

- "Another project may be to find out more about Pythagoras, about his life and about the pythagoreans' discoveries in mathematics. What did they find out about mathematics and music? Why did they have the slogan 'Everything is number'?" This is an example of a project that actually gives an idea of what the pupils will be working with. Far too often textbooks give ideas such as "a project about Pascal", with the danger that pupils will be spending the next few lessons studying what he meant about religion or programming. (Not that those subjects are not interesting – my point is just that the teacher should have an idea of what he wants his pupils to study).

As we see, some exercises are meaningless, some are meaningful. I hope that only the meaningful will survive into the next edition…

Conclusion: Missed opportunities

The main impression after studying these textbooks is not that everything is terrible, but that there are lots of missed opportunities there. One goal for the following years will be to try to point out some of these opportunities, and hope that someone will take advantage of them.

Contact information

I would appreciate ideas on how to include/use history of mathematics in mathematics education. I would also find discussions on this subject interesting. Therefore, I include my addresses:

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