Historical topics in Norwegian textbooks

BJØRN SMESTAD, FINNMARK UNIVERSITY COLLEGE

When the new national curriculum for elementary schools in Norway included history of mathematics in the mathematics curriculum in 1997, I decided to take a look at the new textbooks to see how history of mathematics was incorporated. The study is partly quantitative, but most of it is qualitative, looking at the textbooks from the point of view of the different reasons that are often given for including history of mathematics in mathematics education. My main conclusion is that there are lots of opportunities, but that the textbooks misses most of them.

In Norway, children attend school from the year they turn six, and the first ten years of school are obligatory. Then almost all go on to high schools for three years. There are lots of different high schools to choose from – from ”purely” vocational ones (for pupils who want to become electricians, car mechanics etc. etc.) to ”purely” theoretical ones (for pupils who want to go on to study in colleges or universities).

Both for the first ten years and for the high schools, the government has put down fairly detailed curriculums.

In this article, I will give some comments on my study of Norwegian textbooks for grades 1-10, and also write some words on a study in progress on the textbooks for the first year of high school.

Norwegian textbooks for grades 1-10 (ages 6-16)

History of mathematics was included in the curriculum for the elementary schools in Norway in 1997. I had a look at the textbooks published after this. (The total number of textbook pages studied was about 21.000.[1]) I found that

- history of mathematics occupied only about 1–2 % of the textbooks[2]
- there were lots of factual errors[3]
- some textbooks tended to treat almost only numeral systems
- there were no agreement among the authors on what should be included (no “canon”).

I have also considered whether the textbooks try to motivate the pupils to ask questions on the development of mathematics. There are no clear examples of this. On the contrary, the treatments of numeral systems are

---

[1] This number includes all the different publishers’ sets of textbooks, and also includes teacher’s books.

[2] The average pupil will see about 36 pages of history of mathematics in the ten years combined.

[3] I go into some detail on this in my presentation from ICME 9, see Smestad (2000a).

terrible counter examples – the textbooks tend to present the numeral systems with no further discussion, and then ask the pupils to calculate with them.

Do the textbooks give the pupils leads as to where they can find answers to such questions? Extremely rarely. There are only a handful of such examples in the textbooks.

Reasons for including history of mathematics in mathematics education

There has been given a lot of reasons for including history of mathematics in mathematics education, and I have summarised them into the following list. I have analysed the textbooks from the point of view of each of these reasons.

History of mathematics can explain why we use the definitions, names and symbols we do.

The definitions and names of measuring units often are connected with each other and with the concept of measuring. This is described in some textbooks. How the *meter* is (and has been) defined, is also discussed.

Some words are discussed: ”geometry”, ”Pythagorean theorem”, ”algebra”, ”roman” numeral, ”Celsius”, ”Fahrenheit”, ”Kelvin”, ”million” and the names of the months. I think it makes sense to spend a few words on this in the textbooks.

In addition, theories are offered on how the hindu-arabic numerals and the roman numerals got their apprearances. π and many other symbols are also discussed. Sadly, the theories offered are often wrong. In other cases only the name of a mathematician and a year of first appearance is given. These are often at best inaccurate. Even when correct, these pieces of information may not be very helpful to the pupils.

History of mathematics can explain how formulas were developed.

There are no examples of this in the textbooks. Most of the formulas in elementary school textbooks are probably too old for us to know how they were developed, but some examples of old proofs (for instance of Pythagorean theorem) could be included.

History of mathematics can show pupils a multitude of algorithms, thereby improving their understanding of their own algorithm.

By seeing alternative algorithms, pupils may possibly look at their own algorithm in new ways. In the textbooks, Egyptian multiplication, abacus calculation, regula falsi, regula de tri and Heron’s method for finding square roots are mentioned. I think that for instance al-Khwarizmi’s method of solving certain equations of second degree, could also have been included.

On the other hand, some of the textbooks present exercises with (for instance) Egyptian numerals and our (modern) symbols for addition and subtraction, and ask the pupil to calculate, without mentioning that the Egyptians used other devices than pen and paper to calculate. Thereby, pupils get the impression that Egyptians did calculations the same way that we do, and that their symbols were poorly suited for this, making calculations very difficult. (Obviously, the pupils are not told the etymology of the word *calculate*…)[4]

History of mathematics can show pupils how concepts have developed, and thereby connecting concepts.

Some textbooks point to the connection between number systems and the time unit system, and between numbers and geometry. While not exactly explaining how they developed, at least they show that more of the concepts in mathematics are connected than is apparent at first look.

History of mathematics can give pupils the opportunity to see contrasts between different concepts.

The number concept of (learned) people two thousand years ago was different from our own.[5] By studying this, one may get a different view of our own concept. Other examples are the function concept, unit fractions and the angle concept. I have not found any examples of this in the textbooks.

History of mathematics can give pupils the opportunity of comparing old and new methods, thereby giving the opportunity of choosing the best method in a given situation.

(See algorithms above.)

History of mathematics can explain the role of mathematics in society (give examples where mathematics has been important).

The story of Florence Nightingale is a good example of this. In one of the textbooks, Florence Nightingale's motivation for doing mathematics is focused. Similarly, many other mathematicians have practical reasons for doing mathematics, but these reasons are seldom seen in the textbooks. Instead, the role of mathematics in society is often connected to art and mysticism.

Moreover, mathematicians are not the only ones inhabiting the history of mathematics – maybe the people who have used mathematics should also be included in the textbooks?

To be fair: there are things to find in the textbooks. Some examples are the role of geometry (especially in measuring land), probability theory, counting, equations, architecture and statistics.

---

[4] calculate – from latin *calculus* "reckoning, account," originally "pebble used in counting."

[5] This is both because we learn how to calculate with other classes of numbers (such as decimal numbers, irrational numbers and complex numbers), but also because we more often meet very large and very small numbers (such as billions (of dollars) or millionth part of an inch). This does not, of course, mean that all people today necessarily have a good grasp on what they are doing with these numbers.

History of mathematics can show that mathematics is a result of the work of generations. Mathematics is dynamic, not static.

When reading the textbooks, will the pupils get the impression that mathematics was invented (and finished) by some intelligent old men thousands of years ago, or will they see that mathematics has been developing for a long period of time, and is still developing? While this is not a major point in any of the textbooks, there are many examples that show that mathematics has developed over time.

While specialised, wise men have been important in developing mathematics, other people have also contributed. It seems to me that the people who have used the mathematics, and the children who have been taught it in school, also have a place in the history of mathematics. They should not be completely ignored in the textbooks.

History of mathematics can show that difficulties are a natural part of development.

I think it is important that pupils see that mathematicians have struggled to get to the point where they are today. Seeing concrete examples of these struggles may make it easier for pupils to accept that they also have to struggle a little – not understanding something at once is not a sign that you are stupid…

I have only found three examples of this in the textbooks: the role of zero, the attempts to give the proportion between the circumference and diameter of the circle as accurately as possible, and struggles with the infinity concept. Not only are three examples not much, textbooks also often say that mathematicians "found" or "discovered" something, apparently totally by chance. I think pupils would enjoy seeing what problems mathematicians had, and what were there motivations for spending so much time trying to develop mathematics.

Probability theory is an example of an area where such problems could easily be included. Other areas are perspective, negative numbers and irrational numbers.

History of mathematics can give mathematics a human face.

There are examples of the textbooks telling more about the mathematicians than just their names. However, in all the 10 grades, and all the different publishers' textbooks combined, there are only 19 examples of this. Given that these examples also include a number of errors, I think I can safely say that this situation has potential for improvement.

As this is the Abel-Fauvel Conference, I will include (in full) what one of the textbooks says about our friend Abel:

> *Niels Henrik Abel (1802-1829), one of the world's greatest mathematicians. Was appointed professor in Berlin. Collected works published in French 1839 and 1881.*

There are more interesting facts on Abel than these – as those who attended Ivar Salvesen's talk on Abel (at this conference) will know. Likewise, there are important and interesting information available on most other mathematicians – some of which should be included in textbooks.

History of mathematics can increase the respect of earlier cultures.

Throughout the centuries, mathematics has been an important part of every society's development. When looking at impressive cultural articfacts, we can almost always find that mathematics is involved. Therefore, it's not unreasonable to believe that we should be able to present history of mathematics in a way that increases our understanding and respect of earlier cultures.

For instance the pyramids are treated quite well in some of the textbooks. But this is almost the only example of this that can be found. I think there should be more focus on what the different cultures achieved because of their mathematical insights. The instances where non-European mathematics is mentioned are fairly few (and usually concern numeral systems). I find this unfortunate.

History of mathematics can give pupils experience in using sources, libraries and Internet and in writing essays.

There are lots of ideas for project work in the textbooks. I would wish that they were more "helpful" – giving pupils hints on where to look and what to look for. There are too many ideas of this kind:

> *Use the internet or encyclopaedias to find information on Pierre de Fermat.*

The other extreme is not much better:

> *Use the internet, encyclopaedias or CD-ROM to find out when Johann Widmann lived.*

As the text already stated that he was alive in 1489, it must be the exact years of birth and death that are of interest here.

What is needed, is project ideas that have a better chance of being fruitful than what a random teacher can think of himself.

History of mathematics can give opportunities for crosscurricular work.

There are a number of ideas for crosscurricular projects involving history of mathematics. These are: "Post-WWII – the part of geometry in rebuilding Europe", "the calendar", "one day with the Aztecs", "fractions through history", "old measuring units", "roman numerals", "Archimede", "symmetry", "geometric forms" and "history of the pyramids".

Conclusion

My main conclusion is that there are lots of opportunities, but that this "first generation" of textbooks misses most of them. Occationaly, a textbook includes history of mathematics in a meaningful way in one context, but other textbooks treating the same subject have not had the chance to copy the good idea. On the other hand, this means that improving the textbooks should not be an impossible task.

Norwegian textbooks for the first year of high school

History of mathematics has been part of the curriculum in the obligatory course in the first year of high school for many years. It can be argued that including history of mathematics in high schools is simpler than in the elementary school, both because of the age of the pupils and the topics discussed. Therefore, one might hope that the textbooks for the first year of high school include more history of mathematics and in a better way than in elementary schools.

So far, I have only noticed that the quantity of history of mathematics is larger – 5,7 % (ranging from 2,5 % to 16,2 %). It also seems that there are fewer errors and that the history is better connected to the mathematics studied. However, some of the textbooks place the history in the beginning or in the end of the chapters, and I will consider further the consequences of this.

References

Smestad, Bjørn. (2000a). History of mathematics in Norwegian textbooks (ICME 9, Tokyo). In Smestad, Bjørn: *Matematikkdidaktiske småskriv 1999-2001*. Alta: Høgskolen i Finnmark.

Smestad, Bjørn. (2000b). The introduction of mathematics in Norwegian school books (HME 2000, Taipei). In Smestad, Bjørn: *Matematikkdidaktiske småskriv 1999-2001*. Alta: Høgskolen i Finnmark.

Smestad, Bjørn. (2002). *Matematikkhistorie i grunnskolens lærebøker: en kritisk vurdering*. Alta: Høgskolen i Finnmark.

(All of these are on the elementary school textbooks. I have not yet published anything on the high school textbooks.)